Astron. Nachr./AN **xxx** (xxxx) x, xxx–xxx

QPOs in microquasars and Sgr A*: measuring the black hole spin

G. TÖRÖK[1,2]

[1] Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, 746 01 Opava, Czech Republic
[2] Nordita, Blegdamsvej 17, 2100 Copenhagen, Denmark

Receivedv 8 September 2005; accepted 20 September 2005; published online 20 October 2005

Abstract. In all four microquasars which show double peak kHz QPOs, the ratio of the two frequencies is 3:2. This strongly supports the suggestion that twin peak kHz QPOs are due to a resonance between some modes of accretion disk oscillations. Here, we stress that fits to observations of the hypothetical resonances between vertical and radial epicyclic frequencies (particularly of the parametric resonance) give an accurate estimate of the spin for the three microquasars with known mass. Measurement of double peak QPOs frequencies in the Galaxy centre seems also to be consistent with the 3:2 ratio established by previous observations in microquasars, however the Sgr A* data are rather difficult for the same exact analysis. If confirmed, the 3:2 ratio of double peak QPOs in Sgr A* would be of a fundamental importance for the black hole accretion theory and the precise measurement could help to solve the question of QPOs nature.

Key words: black hole physics – X-ray variability

©0000 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

arXiv:astro-ph/0510669v1 23 Oct 2005

1. Orbital resonance models – the brief sketch

Kluźniak & Abramowicz (2000), motivated by the observed double peak kHz QPOs in X–ray variability of neutron stars sources, introduced models based on the idea of a resonance between the relativistic epicyclic frequencies [1] . These models were reviewed, e.g., in Abramowicz & Kluźniak (2004); Török et al. (2005) or Kluźniak (2005, this textbook).

According to the resonance hypothesis, the two modes in resonance have eigenfrequencies ν_{rad} (equal to the radial epicyclic frequency) and ν_{v} (equal to the vertical orbital frequency ν_θ or to the Keplerian frequency ν_{K}). Several resonances of this kind are possible, and have been discussed (see, e.g., Abramowicz & Kluźniak 2004). Main relations between the eigenfrequencies of resonance and the possibly observable frequencies are summarized in Table 1 for some particular resonances.

Correspondence to: terek@volny.cz

[1] Before the first discovery of the double peak QPO in black hole source GRO 1655–40 by Strohmayer (2001), Kluźniak & Abramowicz (2000) suggested on theoretical ground that such eventual QPOs should have rational ratios.

Table 1. Relation of observed frequencies for standard ($\nu = \nu_\theta$) and "Keplerian" ($\nu = \nu_{\mathrm{K}}$) resonances – while the parametric model identifies the observed ν_{up}, ν_{down} directly with the eigenfrequencies of resonance, forced resonances can give the 3:2 ratio by combinational frequencies.

	Type of resonance ($\mathrm{n}\nu_{\mathrm{rad}} = \mathrm{m}\nu$)	n	m	Observed frequencies ν_{upp}	ν_{down}
standard	parametric	3	2	ν_θ	ν_{rad}
	3:1 forced	3	1	ν_θ	$\nu_\theta - \nu_{\mathrm{rad}}$
	2:1 forced	2	1	$\nu_\theta + \nu_{\mathrm{rad}}$	ν_θ
Keplerian	parametric	3	2	ν_{K}	ν_{rad}
	3:1 forced	3	1	ν_{K}	$\nu_{\mathrm{K}} - \nu_{\mathrm{rad}}$
	2:1 forced	2	1	$\nu_{\mathrm{K}} + \nu_{\mathrm{rad}}$	ν_{K}

2. Estimating the black hole spin

Formulae for the Keplerian and epicyclic frequencies ν_{vert} and ν_{rad} in the gravitational field of a rotating Kerr black hole with the mass M and internal angular momentum a (here and henceforth *spin* a) are well known (Fig. 1) – see, e.g., Nowak & Lehr (1999):

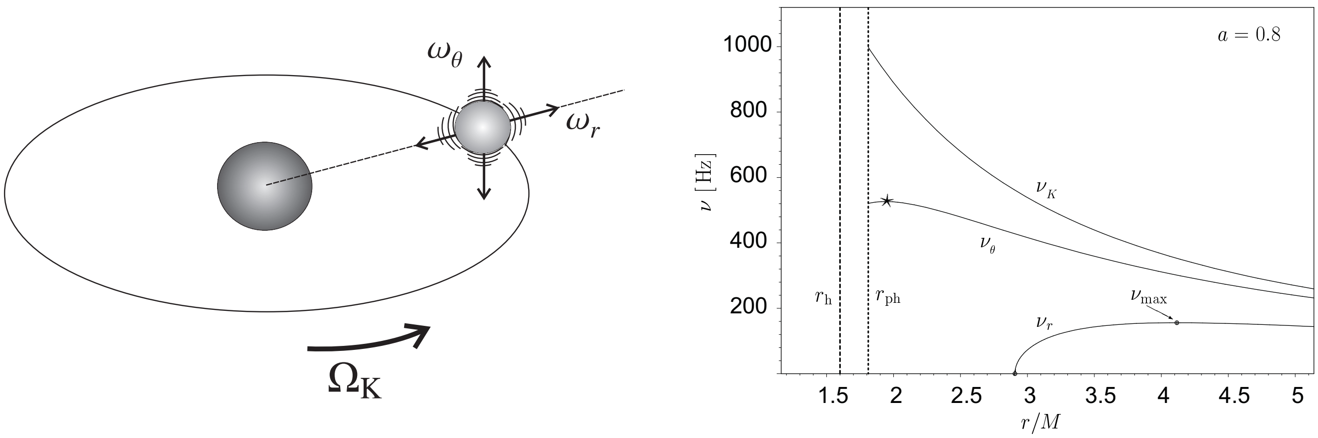

Fig. 1. *Left cartoon:* In the gravitational field of a central object, test particle on a circular orbit starts oscillate after the small perturbation. Frequencies of these oscillations (radial ν_r and vertical ν_θ) are fundamentally different in Newton's and Einstein's gravity. In Newtonian physics these epicyclic frequencies must always be equal to the Keplerian frequency of circular orbit and the resulting trajectory is an ellipse, while in Einstein's theory they differ and trajectory is not closed. *Right:* Figure plotted for moderately rotating $10\,\mathrm{M}_\odot$ black hole shows behaviour of epicyclic frequencies typical for Kerr black holes – strong Einstein's gravity makes $\nu_K \geq \nu_\theta > \nu_r$.

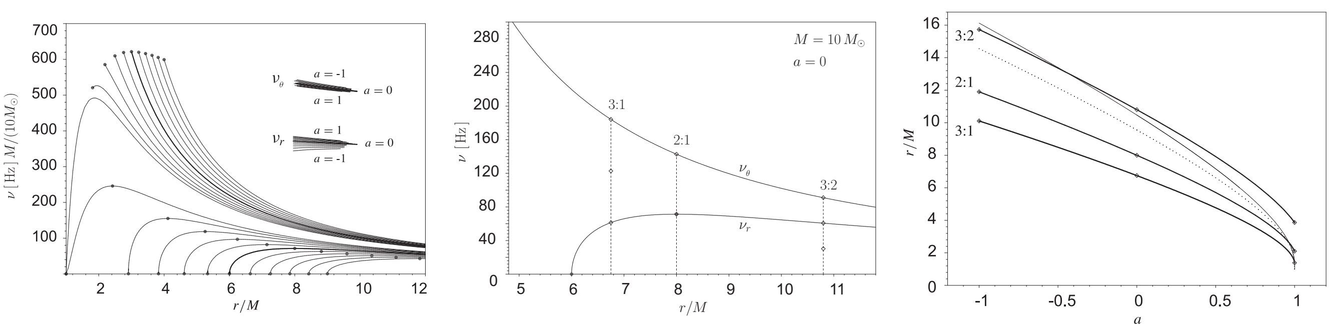

Fig. 2. *Left:* Behaviour of epicyclic frequencies is sensitiv to the spin. Here, the curves are spaced every 0.2 in a. *Middle:* Locations of the three particular resonances for the Schwarzschild black hole ($a = 0$). *Right:* Locations of the same three resonances as functions of the black hole spin (discontinuos lines indicate the position of main part of the accretion disc for two idealized cases of thin and thick disc).

$$
\begin{aligned}
\nu_K &= \frac{1}{2\pi}\left(\frac{GM_0}{r_G^3}\right)^{1/2}\left(x^{3/2}+a\right)^{-1}, \\
\nu_{rad}^2 &= \nu_K^2\left(1-6\,x^{-1}+8\,a\,x^{-3/2}-3\,a^2\,x^{-2}\right), \qquad (1)\\
\nu_{vert}^2 &= \nu_K^2\left(1-4\,a\,x^{-3/2}+3a^2\,x^{-2}\right),
\end{aligned}
$$

where $x = r/(GM/c^2)$ is the dimensionless radius expressed in terms of the gravitational radius of the black hole.

For a particular resonance n:m, the equation

$$
\mathrm{n}\nu_{rad} = \mathrm{m}\nu; \quad \nu = \nu_\theta \text{ or } \nu_K \qquad (2)
$$

determines the dimensionless resonance radius $x_{n:m}$ as a function of spin a (see Fig. 2) [2]. Thus, from the observed frequencies and from the estimated mass one can calculate the relevant spin of a central black hole (Abramowicz & Kluźniak 2001; Török et al. 2005).

[2] Because of the properties of Kerr black hole spacetimes, *any* relativistic model of black hole QPOs should be rather sensitive to the spin a, however this sensitivity can be negligible on large scales of mass (Abramowicz et al. 2004a) – see Fig. 5.

2.1. Microquasars observational data

Till this time, double peak kHz QPOs were measured in the case of four microquasars (GRO 1655–40; GRS 1915+105, XTE 1550–564, H 1743–322) and in every case the data show 3:2 ratio of frequencies in the double peak ($\nu_{upp}/\nu_{down} = 3/2$). Mass estimates for central black holes are known and moreless firmly established for the three of these four sources, however for the GRS 1915+105 the mass estimate still varies of factor about two (Greiner, Cuby, McCaughrean 2001) and for GRO 1655–40 exists two incompatible studies of the mass (Greene, Bailyn, Orosz 2001; Beer & Podsiadlowski 2002) – for the summary of microquasars data see Table 2 together with its references, while Fig. 3 shows the illustration of significant 3:2 ratio.

2.2. The Sgr A* data

The mass of Galaxy centre black hole have been discussed in several recent studies and the best estimate is usually given as $(3.6 \pm 0.4) \times 10^6\ \mathrm{M}_\odot$ (e.g. Schoedel et al. 2002, 2003;

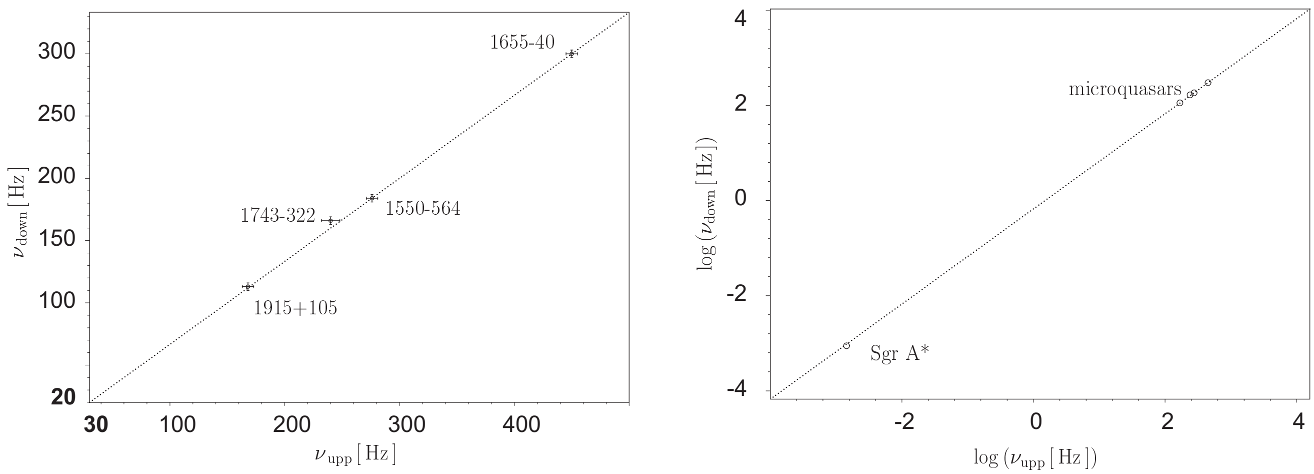

Fig. 3. *Left:* In all four microquasars where double peak kHz QPOs were detected, the observed frequencies $\nu_{\rm upp}$ and $\nu_{\rm low}$ are clearly in the 3:2 ratio. *Right:* The same 3:2 ratio seems to be present in double peak QPOs in Sgr A*.

Table 2. Frequencies of twin peak QPOs in microquasars and Galaxy centre black hole

Source [(a)]	$\nu_{\rm upp}$ [Hz]	$\Delta\nu_{\rm upp}$ [Hz]	$\nu_{\rm down}$ [Hz]	$\Delta\nu_{\rm down}$ [Hz]	$2\nu_{\rm upp}/3\nu_{\rm down} - 1$	Mass [(b)] [$M_\odot$]
GRO 1655–40	450	± 3	300	± 5	0.00000	6.0 — 6.6
XTE 1550–564	276	± 3	184	± 5	0.00000	8.4 — 10.8
H 1743–322	240	± 3	166	± 8	-0.03614	not measured
GRS 1915+105	168	± 3	113	± 5	0.00885	10.0 — 18.0
Sgr A*	1.445	$\pm$0.16 mHz	0.886	$\pm$0.04 mHz	0.08728	(2.6 — 4.4) $\times 10^6$

[(a)] Twin peak QPOs first reported by Strohmayer (2001); Remillard et al. (2002); Homan et al. (2003); Remillard et al. (2003); Aschenbach et al. (2004).
[(b)] See Greene, Bailyn, Orosz (2001); Orosz et al. (2002); Greiner, Cuby, McCaughrean (2001); McClintock & Remillard (2003) for the microquasars. Note that there is the different estimate for GRO 1655–40: $M = (5.4 \pm 0.3)\,\mathrm{M}_\odot$ (Beer & Podsiadlowski (2002)). Interval for Sgr A^* (used in Török et al. 2005) is resulting from several recent analysis.

Eisenhauer et al. 2003; Ghez et al. 2003, 2004); however substantially lower mass is not excluded yet (Reid et al. 1999, 2003; Backer & Sramek 1999; Schoedel et al. 2003).

Aschenbach et al. (2004) have reported five QPOs periodicities in the Galaxy centre black hole X–ray variability: 692 sec, 1130 sec, 2178 sec, 100 sec and 219 sec. Shortly before the Aschenbach's measurement, Genzel et. al. (2003) found a clear periodicity of 17 min (1020 sec) in Sgr A* infra-red variability during a flaring event. It is rather difficult to confirm the Aschenbach's data, nevertheless note that (Abramowicz et al. 2004a,b; Aschenbach 2004; Török et al. 2005):

$$(1/692) : (1/1130) : (1/2178) \approx 3 : 2 : 1.$$

2.3. Estimates of the spin from particular models (quantitative results)

Angular momentum estimates are shown in Table 3 for several particular resonance models, while Fig. 4 illustrates how the 3:2 parametric resonance model fits the data. For the Sgr A^*, we present the estimate of spin in Fig. 5 (right panel).

3. Discussion and conclusions

In the case of microquasars all resonances discussed here (except the 3:2 Keplerian resonance) are consistent with the existing microquasars data and comparing this data with predictions of the theory gives a clear estimate of the spin for given source and model. The different relativistic QPOs models can be compared at the moment when some independent and convincing estimate of the microquasars spin will be done. For example, the 3:2 parametric resonance model [3] expects high spin ($a \sim 0.95$) for the microquasars, while another relativistic precession model of Morsink & Stella (1999) predicts the spin to be rather low ($a \sim 0.2$). Unfortunately, no such clear spin predictions exist at the present stage. For example, it is often argued that the presence of relativistic jet (which the discussed microquasars show) is a signature of large black hole spin (Blandford & Znajek 1977) but there is some evidence against (Ghosh & Abramowicz 1997). Also the up-and-coming studies of spectral iron lines give contradictory results as well (see, e.g., Martocchia et al. 2002).

[3] One should remind that, in difference to forced resonances, the 3:2 parametric resonance model gives the 3:2 observed ratio naturally without any additional assumptions.

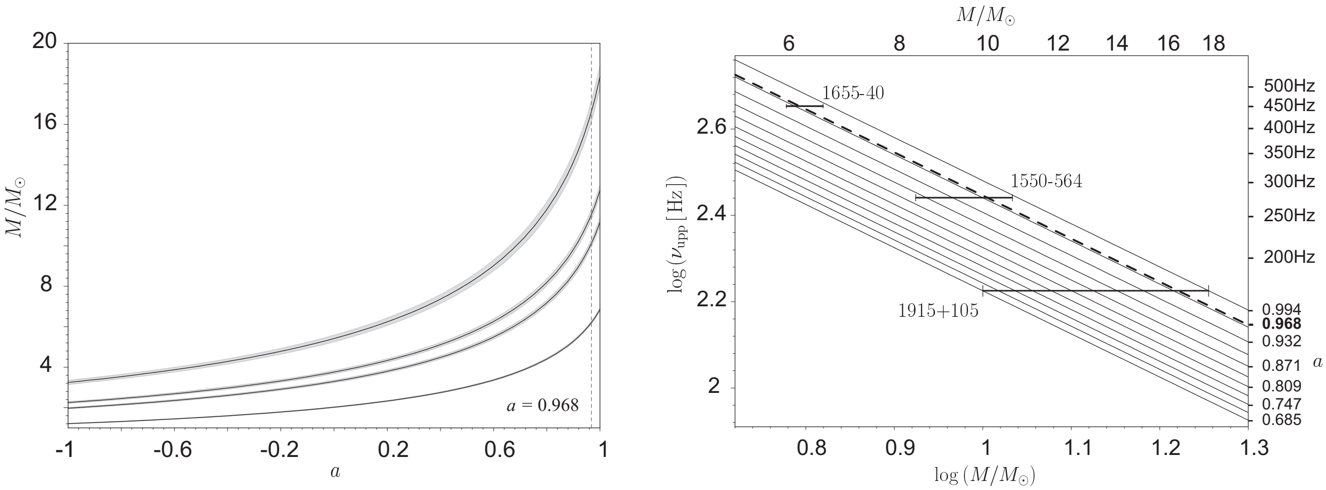

Fig. 4. *Left:* Mass-spin dependence $M(a)$ calculated from the 3:2 parametric resonance model for the frequencies $\nu_{\rm upp} =$ 168 Hz, 242 Hz, 276 Hz, and 450 Hz (from the top in this order) observed in four microquasars. Shadows show the range ± 5 Hz. *Right:* the same in the form of fit to the observational data. For both panels, the dashed line $a = 0.968$ corresponds to the observational fit $\nu_{\rm upp} = 2.793\,{\rm M}_\odot/M$ kHz found by McClintock & Remillard (2003) .

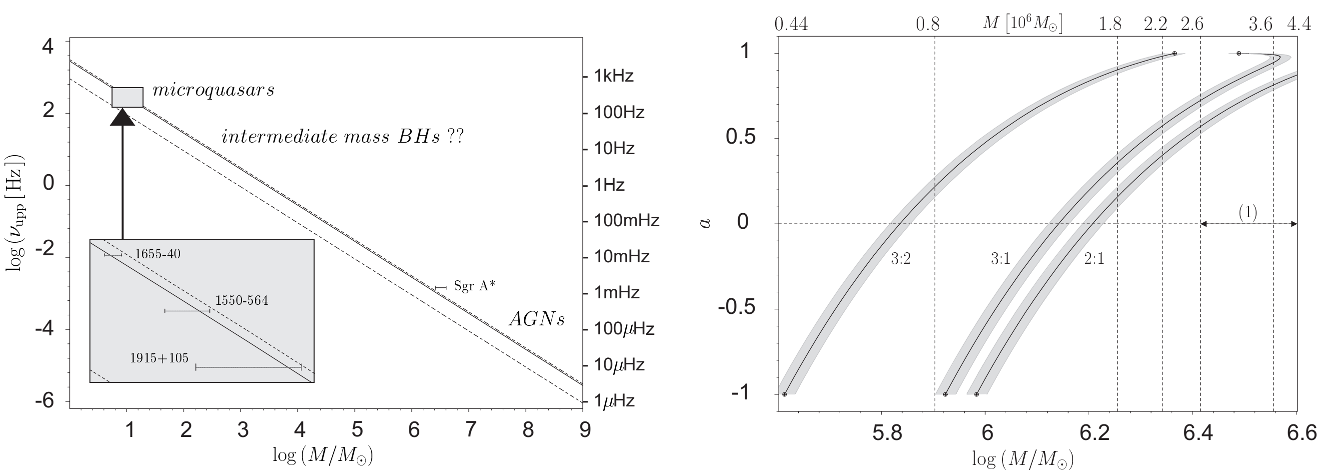

Fig. 5. *Left:* The upper frequency of the double peak predicted as the function of mass by 3:2 parametric resonance model. Mass range for Sgr A* corresponds to the range given in Table 1. Dotted lines are for $a = 0$ (lower line) and $a = 1$ (upper line), the solid line is the fit $1/M$ found by McClintock & Remillard (2003). *Right:* Spin dependence for 3:2 parametric, 3:1 and 2:1 forced resonance in Sgr A* implied by $\nu_{\rm down} = 0.886$ mHz measured by Aschenbach et al. (2004), shadows respect accuracy states in that measurement. For the 3:1 forced resonance and very high spin, the estimate is ambigous what is valid for any epicyclic resonance with $p/q > 2.18$ - see Török & Stuchlík (2005) where the properties of epicyclic frequencies in the Kerr spacetimes are discussed in detail.

In the case of Galaxy centre black hole the spin estimate following from the measurement of the 3:2 QPOs frequencies (1/692):(1/1130) and the most accepted prediction $M_{\rm Sgr\,A^*} \sim 3.6 \times 10^6\,{\rm M}_\odot$ rather exlude the 3:2 parametric resonance and imply moderate black hole internal angular momentum for the eventual 2:1 forced resonance while for the 3:1 resonance the spin would be rather high. But, in fact, it is difficult to discuss this spin predictions seriously – the mass of Sgr A* is known with the accuracy of one order of magnitude in present and the frequency measurement by Aschenbach et al. (2004) is not confirmed yet. So one can expect that in principle no of the resonances considered here is excluded (see Fig. 5).

Finally we stress that the 3:2 ratio seems to be now significant for the compact X-ray sources [4] and the 3:2 data of Aschenbach et al. (2004) strongly supports the idea of relativistic scaling for QPOs frequencies (Abramowicz et al., 2004a) – if confirmed, these would be of a fundamental importance for the black hole accretion theory and the precise measurement can help to solve the question of QPOs nature.

[4] For black holes see references in the Table 2 and van der Klis (2005, this textbook). For neutron stars see, e.g, Abramowicz et al. (2003); Belloni et al. (2005), or Bulik (2005, this textbook); Barret (2005, this textbook). Overview of the data and detailed references are given in van der Klis (2005).

Table 3. Summary of angular momentum estimates from resonance models for microquasars

		Interval of possible spin a relevant for			
Model for		1550–564	1655–40	1655–40*	1915+105
3:2 $[\nu_\theta,\ \nu_r]$	parametric	+0.89 — +0.99	+0.96 — +0.99	+0.88— +0.93	+0.69 — +0.99
2:1 $[\nu_\theta,\ \nu_r]$	forced	+0.12 — +0.42	+0.31 — +0.42	+0.10— 0.25	-0.41 — +0.44
3:1 $[\nu_\theta,\ \nu_r]$	forced	+0.32 — +0.59	+0.50 — +0.59	+0.31— +0.44	-0.15 — +0.61
3:2 $[\nu_\mathrm{K},\ \nu_r]$	"Keplerian" p.				+0.79
2:1 $[\nu_\mathrm{K},\ \nu_r]$	"Keplerian" f.	+0.12 — +0.43	+0.31 — +0.42	+0.10— +0.25	-0.41 — +0.44
3:1 $[\nu_\mathrm{K},\ \nu_r]$	"Keplerian" f.	+0.29 — +0.54	+0.45 — +0.53	+0.28— 0.40	-0.13 — +0.55

Table contains values of dimensionless spin calculated exactly from the upper observed frequency, error Δa resulting from uncertainty of frequency measurement $\Delta\nu_\mathrm{upp}$ is for microquasars XTE 1550-564 (GRO 1655-40, GRS 1915+105) given as $\sim\pm 0.03$ (0.01, 0.05).

* This second column for GRO 1655–40 shows numbers outgoing from the mass analysis by Beer & Podsiadlowski (2002). Theirs estimate $M_{\mathrm{\ GRO\ 1655-40}} \sim 5.4\,\mathrm{M}_\odot$ is nearly one solar mass lower than that by Greene, Bailyn, Orosz (2001). Note that while the estimate of spin from 3:2 parametric resonance is for both cases (6.3; $5.4\ M_\odot$) high and in fact similar: $a \approx 0.9$, for other discussed eventualities this difference in the mass results in large difference of spin.

Acknowledgements. I thank Marek Abramowicz, Wlodek Kluzniak and Zdenek Stuchlik for discussion and help. This work was supported by the Czech grant MSM 4781305903. I also thank the excellent hospitality of Nordita (Copenhagen).

References

Abramowicz, M.A., Bulik, T., Bursa, M. & Kluźniak, W. 2003, A&A 404, L21
Abramowicz, M.A., Kluźniak, W. 2001, A&A 374, L19
Abramowicz, M.A., Kluźniak, W., McClintock, J.E & Remillard, R.A. 2004a, ApJ 609, L63
Abramowicz, M.A. & Kluźniak, W. 2004, in: P. Kaaret, F.K. Lamb, J.H. Swank (eds.), *X-Ray Timing 2003: Rossi and Beyond*, AIP Conf. Proc. 714, American Inst. of Physics, Melville (NY)
Abramowicz, M.A., Kluźniak, W., Stuchlík, Z. & Török, G. 2004b, in: S. Hledík, Z. Stuchlík (eds.), Proceedings of RAGtime 4/5: Workshops on *Black holes and neutron stars*, Silesian University Opava
Aschenbach, B., Grosso, N., Porquet, D. & Predehl, P. 2004, A&A 417, 71
Aschenbach, B. 2004, A&A 425, 1075
Backer, D.C. & Sramek, R.A. 1999, ApJ 524, 805
Barret, D. 2005, AN 326, 808
Beer, M.E. & Podsiadlowski, P. 2002, MNRAS 331, 351
Belloni, T., Mendez, M. & Homan, J. 2005, A&A 437, 209
Blandford, R.D. & Znajek, R.L. 1977, MNRAS 179, 433
Bulik, T. 2005, AN 326, 861
Eisenhauer, F., Schödel, R., Genzel, R. et al. 2003, ApJ 597, L121
Genzel, R., Schödel, R., Ott, T. et al. 2003, ApJ 594, 812
Ghez, A.M., Duchêne, G., Matthews, K. et al. 2003, ApJ 586, L127
Ghez, A. M., Salim, S., Hornstein, S. D., Tanner, A., Lu, J.R., Morris, M., Becklin, E.E. & Duchene, G. 2004, astro-ph 0306130
Ghosh, P., Abramowicz, M.A. 1997, MNRAS 292, 88
Greene, J., Bailyn, Ch.D. & Orosz, J.A. 2001, ApJ 554, 1290
Greiner, J., Cuby, J.G. & McCaughrean, M.J. 2001, Nature 414, 522
Homan, J., Miller, J.M., Wijnands, R., Steeghs, D., Belloni, T., van der Klis, M. & Lewin, W.H.G. 2003, Atel 16 http//integral.rssi.ru/atelmirror/
Kluźniak, W. & Abramowicz, M.A. 2000, PhRvL, submitted (astro-ph/0105057)
Kluźniak, W. & Abramowicz, M.A. 2003, *12th Workshop on General Relativity and Gravitation*, Tokyo Univ. Press, Tokyo (astro-ph/0304345)
Kluźniak, W. 2005, AN 326, 820
McClintock, J.E. & Remillard, R.A. 2003, astro-ph/0306213 v.2
Martocchia, A., Matt, G., Karas, V. % Feroci, M. 2002, A&A 387, 215
Morsink, M. & Stella, L. 1999, ApJ 513, 827
Nowak, M. & Lehr, D. 1999, in M.A. Abramowicz, G. Björnsson, J.E. (eds.), *Theory of Black Hole Accretion Disks*, Cambridge Univ. Press, Cambridge
Orosz, J.A., Groot, P.J., van der Klis, M. et al. 2002, ApJ 568, 845
Remillard, R.A., Muno, M.P., McClintock, J.E. % Orosz, J.A. 2002, ApJ 580, 1030
Remillard, R.A., Muno, M.P., McClintock, J.E. & Orosz, J.A. 2003, AAS HEAD meeting 7, 30.03
Reid, M.J., Readhead, A.C.S., Vermeulen, R.C. & Treuhaft, R.N. 1999, ApJ 524, 816
Reid, M.J., Menten, K.M., Genzel, R. et al. 2003, ANS 324, 505 (astro-ph/0304095)
Schödel, R., Ott, T., Genzel, R. et al. 2002, Nature 419, 694
Schoedel, R., Ott, T., Genzel, R. et al. 2003, ApJ 596, 1015
Strohmayer, T. 2001, ApJ 552, L49
Török, G., Abramowicz, M.A., Kluźniak, W. & Stuchlík, Z. 2005, A&A 436, 1
Török, G. & Stuchlík, Z. 2005, A&A 437, 775
van der Klis, M. 2004, astro-ph/0410551
van der Klis, M. 2005, AN 326, 798